Theoretical expressions of thermal conductivity of wood

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Abstract: In this paper, the theoretical expressions of wood thermal conductivity in the choral and radical direction are derived from the micro–structure of wood by applying some basic principles in physical mechanics. The thermal conductivities of about twenty species of trees were calculated by means of the expressions and compared with its experimental values under the same condition. The average relative error is about 5%, so the calculation result is satisfactory.

Key words: Thermal conductivity; Wood; Choral direction; Radical direction; Theoretical expression

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Introduction

Thermal conductivity is required in scientific research and heat treatment of wood. Because wood is a kind of natural polymer organism whose chemical element and microstructure are very complicated, it's very difficult to study its thermal conductivity theoretically. Therefore, most scholars at home and abroad study presently the thermal conductivity by means of the experiments (Cheng *et al.* 1985; Kollmann *et al.* 1968; Gao *et al.* 1993). Various empirical equations calculating the thermal conductivity of wood have been put forward based on the experimental data. But the question is that the empiric equations of the wood thermal conductivity derived from experimental data according to mathematics fitting have different functions, different adaptable range (species trees, moisture content, temperature and so on), with a larger errors, and lacking the theoretical foundation. In order to solve the above-mentioned problems, the universal theoretical expressions of wood thermal conductivity are derived in this paper in terms of basic principles of physical mechanics and wood physics.

Mathematics deduction of thermal conductivity of wood

The carrier of energy in heat conduction of the general insulation solid is the transversal wave and longitudinal wave, i.e. the acoustic wave, caused by the elastic vibration of the crystal lattice. The lattice vibration shows wave-particle duality.

In order to derive the theoretical expressions of thermal conductivity of wood, we may assume that the phonon is the carrier of energy in heat conduction of wood. The phonon shows the particle property of lattice wave, and moves irregularly in all directions in wood at sound velocity of u as gas molecules do, colliding each other and exchanging their energy. A phonon with velocity u, total energy E and mean free path L can transport energy E when it passes through the distance of L.

Considering the double direction movement of various phonons, we find that energy is transported from the region of larger E to the one of smaller E, that is, from the region with higher temperature to the region with lower temperature.

Now let's consider the number of the phonons which penetrating vertically through one plane unit area S located in x=x_0, which is shown in Fig. 1.

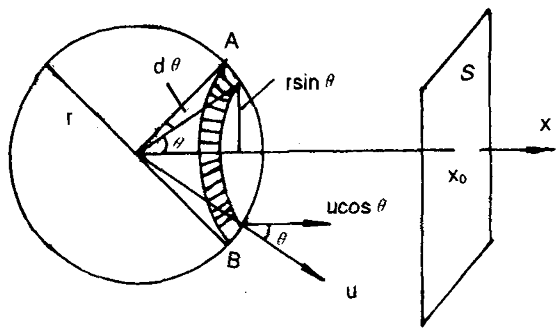

Fig.1 The derivation of thermal conductivity

Supposing that the velocity u of the phonon remains unchanged and the number density of phonon per unit volume is n. The result of a statistical analysis shows that the number of phonons passed within the solid angle dΩ(corresponding to the angle θ and

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$\theta + \mathrm{d}\theta$ relative to the direction of ox axis, shown as a shaded ring in Fig.1) is in proportion to $2\pi(r\sin\theta)r\mathrm{d}\theta$ (i.e. the area of round ring AB). Therefore the number of phonons penetrating the unit area S per unit time in a small angle interval between the angle θ and $\theta+\mathrm{d}\theta$ is

$$n\frac{2\pi r^2 \sin\theta \mathrm{d}\theta}{4\pi r^2}u\cos\theta = \frac{1}{2}nu\sin\theta\cos\theta\mathrm{d}\theta \quad (1)$$

It is clear that the preceding collision of the phonons which are able to penetrating though the plane S from the left or the right side takes place in $x = x_0 - L\cos\theta$ or $x = x_0 + L\cos\theta$ and then move freely though the plane S located in x=x_0. We shall assume that the average energy of the phonon in x and x_0 is $E(x)$ and $E(x_0)$ respectively. When the value of $L\cos\theta$ is smaller, $E(x)$ may be expressed as following

$$E(x) = E(x_0 \pm L\cos\theta) \approx E(x_0) \pm L\cos\theta\frac{\mathrm{d}E}{\mathrm{d}x} \quad (2)$$

where the positive symbol corresponds to the phonons located in the right of the plane S and the negative symbol corresponds to the phonon located in the left of the plane S.

Multiplying formula (1) by formula (2) we can get the energy of the phonons which remove from the left (or the right) to the right (or the left) of the plane S along the direction between $\theta - \theta+\mathrm{d}\theta$. The integral of the difference in energy of the two ones between $0-\frac{\pi}{2}$ is the net quality of heat Q which transfers from the left to the right through the unit area S in per unit time

$$Q = -nuL\frac{\mathrm{d}E}{\mathrm{d}x}\int_0^{\frac{\pi}{2}}\cos^2\theta\sin\theta\mathrm{d}\theta = -\frac{1}{3}nuL\frac{\mathrm{d}E}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}x} \quad (3)$$

According to the macroscopic law of heat conduction, the quality of heat that transfer though the unit area in per unit time is

$$Q = -\lambda\frac{\mathrm{d}T}{\mathrm{d}x} \quad (4)$$

After comparing equation (3) with equation (4), we obtain

$$\lambda = \frac{1}{3}unL\frac{\mathrm{d}E}{\mathrm{d}T} = \frac{1}{3}uL\frac{\rho_0}{M}C_V \quad (5)$$

where ρ_0 is the mass density of dry wood , M is the molar mass of wood cell, and C_V is the molar heat capacity at constant volume.

According to the above—mentioned mechanism of heat conduction of insulation solid, C_V is just Debye' molar heat capacity, that is

$$C_V = 3RD(\frac{\Theta_D}{T}) \quad (6)$$

where $D(\frac{\Theta_D}{T})$ is Debye' function, Θ_D is Debye' temperature, that is

$$\Theta_D = \frac{hu}{k}\left(\frac{3}{4\pi V}\right)^{1/3} \quad (7)$$

where h is Planck' constant, k is Boltzmann' constant, V is the volume of wood's unit crystal cell, u is the sound velocity in wood.

Substituting (6) into (5) we obtain

$$\lambda = \frac{1}{M}Ru\rho_0 LD\left(\frac{\Theta_D}{T}\right) \quad (8)$$

equation (8) may be used to calculate the thermal conductivity in theory, but it is difficult to decide the mean free path of the phonon L. Generally L is related not only to the direction of heat current, but also to the frequency of sound wave in wood, temperature and density of wood, etc (Qian 1962). Therefore, it is very difficult to decide the expression of the mean free path L of the phonon theoretically. According to the experimental results and theoretical analysis, we assume that there exist the relationship between the mean free path L and temperature T, wood density, Debye' characteristic temperature Θ_D as follows

$$L = C_1\left(\frac{T}{\Theta_D}\right)^{3/2}\exp\left(C_2\sqrt{\frac{\rho_A}{\rho_0}}\right) \quad (9)$$

where ρ_A is density of air (values in normal state), C_1 and C_2 are two constants to be decided (C_1 is of length's dimension), and can be decided according to the experimental data of the thermal conductivity. On the basis of higher precision, we can use C_1 =10.15×10^{-10} m, C_2=7.987 (for heat current in the choral direction), and C_1 =12.60×10^{-10} m, C_2=5.283 (for heat current in the radical direction).

In order to simplify calculation, we can expand Debye' function in expression (8) into Taylor' series at T=273.15 K, and expand $T^{3/2}$ in expression (9) into power series of t. Then, by substituting known quantity, we can obtain respectively simplified for-

mula of wood thermal conductivity in the choral and the radial direction

$$\lambda_{cho} = (9.257 + 0.0519t) \times 10^{-3} \frac{\rho_0}{\sqrt{u}} \exp\left(\frac{9.027}{\sqrt{\rho_0}}\right) \quad (10)$$

$$(\mathrm{W \cdot m^{-1} \cdot K^{-1}})$$

$$\lambda_{rad} = (11.49 + 0.0645t) \times 10^{-3} \frac{\rho_0}{\sqrt{u}} \exp\left(\frac{6.0}{\sqrt{\rho_0}}\right) \quad (11)$$

$$(\mathrm{W \cdot m^{-1} \cdot K^{-1}})$$

Comparison of theoretical values with experimental values

The relationship between thermal conductivity of wet wood (whose moisture content is w%) and that of dry wood is shown as the following equation (Kollmann *et al.* 1968)

$$\lambda_w = (1 + 1.25W\%)\lambda \quad (12)$$

where λ may calculated by means of the equation (10) or (11). It is interesting to note that ρ_0 in equations (10) and (11) is density of absolutely dry wood. In fact, the density ρ_w with different moisture content can be turned into absolutely-dry density ρ_0 by the following equation (Cheng *et al.* 1985).

$$\rho_0 = \frac{1 + 0.01K_V W}{1 + 0.01W} \rho_w \quad (13)$$

where K_v is average volume shrinkage coefficient of wood. Evidently if the sound velocity, density, temperature, moisture content and volume shrinkage coefficient of wood are known, the wood thermal conductivity in the choral or the radial direction can be worked out. We have calculated the thermal conductivity in the choral of 23 species of trees at different temperature and with different moisture content, and compared it with experimental values under the same conditions, as shown in Table 1.

Table 1. Theoretical and experimental values of thermal conductivity in chord direction of 23 species of wood

Species of trees	Moisture content W(%)	Temp. t(℃)	Contract Coefficient K_v	Density ρ_w ($\mathrm{kg \cdot m^{-3}}$)	Sound velocity u($\mathrm{m \cdot s^{-1}}$)	Thermal conductivity ($\mathrm{W \cdot m^{-1}\ K^{-1}}$) Theo.	Expe.	Error (%)
Pinus Koraiensis	11.0	16.5	0.459	456	5057	0.1080	0.0989	+9.2
Cunninghamia lanceolata	17.0	23.5	0.421	459	5079	0.1166	0.1070	+9.0
Pinus massoniana	14.6	24.2	0.486	490	4958	0.1235	0.1221	+1.1
Podocarpus imbricatus	13.6	17.4	0.419	529	4248	0.1351	0.1500	-9.9
Larix olgensis	14.5	13.9	0.554	702	4658	0.1627	0.1605	+1.4
Paulownia fargesii	10.5	17.2	0.334	253	4394	0.0740	0.0721	+2.6
P. fortunei	13.0	21.7	0.320	246	4386	0.0755	0.1756	-0.1
P. catalpifolia	12.7	22.4	0.344	312	4442	0.0892	0.1849	+5.1
P. tomentosa var. inlingensis	11.3	17.6	0.333	321	4360	0.0889	0.1896	-0.8
P. tomentosa	13.0	22.4	0.261	341	4489	0.0943	0.0930	+1.4
Alniphyllum fortunei	13.9	24.5	0.423	450	4609	0.1186	0.1210	-2.0
Tilia mandshurica	10.9	16.0	0.447	470	5008	0.1107	0.0989	+11.9
Juglans mandshurica	14.5	16.1	0.486	481	4973	0.1166	0.1105	+5.5
Catalpa duolouxii	17.3	26.0	0.368	482	4688	0.1271	0.1233	+3.1
Melia azedarach	15.3	24.5	0.413	486	4532	0.1283	0.1314	-2.4
Betula platyphylla	13.2	16.7	0.494	583	4957	0.1354	0.1233	+9.8
Acer davidii	12.3	24.5	0.388	616	4310	0.1552	0.1535	+1.1
Acer mono	11.7	25.0	0.544	659	4471	0.1631	0.1686	-3.3
B. alnoides	15.3	24.2	0.541	674	4879	0.1627	0.1698	-4.2
Fraxinus mandshurica	12.1	26.0	0.548	702	4882	0.1658	0.1524	+8.8
Quercus mongolica	12.7	25.0	0.555	721	4288	0.1809	0.1710	+5.8
Homalium hainanense	12.0	24.0	0.565	900	4494	0.2107	0.1965	+7.2
Quercus acutissima	14.8	21.0	0.578	963	4433	0.2265	0.2396	-5.5

We have also computed the thermal conductivity in the radial direction of 18 species of trees, and compared it with experimental values, as shown in Table 2.

It is clear from Table 1 (the thermal conductivity in the choral direction) that the maximum deviation from the experimental value is 11.9%, and the average error is 4.8%. Simultaneously, it is evident from Table 2 (the thermal conductivity in the radial direction) that the maximum deviation from the experimental value

is 13.7%, and the average error is 5.3%.

It will be seen therefore that the results of the comparison between the theoretical values and the experimental ones of thermal conductivity of various species of wood under different conditions is satisfactory. Therefore, it follows that the theoretical expressions of thermal conductivity in the choral and the radial direction in this paper can basically reflect the objective law of thermal properties of wood.

Table 2. Theoretical and experimental values of thermal conductivity in radial direction of 18 species of wood

Species of trees	Moisture content W(%)	Temp. t(°C)	Contract Coefficient K_v	Density ρ_w (kg•m^{-3})	Sound velocity u(m•s^{-1})	Thermal conductivity (W•m^{-1} K^{-1})		Error (%)
						Theo.	Exper.	
Pinus koraiensis	11.7	16.5	0.459	438	5057	0.1123	0.1163	-3.4
Pinus massoniana	15.0	24.2	0.486	531	4958	0.1428	0.1256	+13.7
Paulownia fargesii	10.2	20.6	0.334	250	4394	0.0757	0.0791	-4.3
P. fortunei	13.3	21.7	0.320	262	4386	0.0805	0.0884	-8.9
P. catalpifolia	12.7	22.4	0.344	327	4442	0.0961	0.1000	-3.9
P. tomentosa var. tsinlingensis	10.6	17.6	0.333	306	4360	0.0884	0.0907	-2.5
P. tomentosa	12.7	22.4	0.261	335	4489	0.0967	0.0942	+2.6
Alniphyllnus fortunei	14.1	24.5	0.423	418	4609	0.1192	0.1314	-9.3
Tilia mandshurica	10.6	16.0	0.447	416	5008	0.1166	0.1128	+3.4
Juglans mandshurica	13.4	16.1	0.486	481	4973	0.1243	0.1279	-2.8
Catalpa duolouxii	12.6	26.0	0.368	472	4688	0.1300	0.1233	+5.4
Melia azedarach	15.1	24.5	0.413	420	4532	0.1457	0.1407	+3.6
Betula platyphylla	12.6	16.7	0.494	596	4957	0.1496	0.1512	-1.0
Acer davidii	11.5	24.5	0.388	589	4310	0.1622	0.1803	-10.0
Acer mono	12.0	25.0	0.544	670	4471	0.1821	0.1942	-6.2
B. alnoides	15.0	24.2	0.541	752	4879	0.1850	0.1954	-5.3
Fraxinus mandshurica	12.1	26.0	0.548	680	4882	0.1776	0.1768	+0.4
Quercus mongolica	11.5	25.0	0.555	637	4288	0.1774	0.1942	-8.6

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